Worksheet 107 Solutions

2006 #4

(a) Average acceleration of rocket A is $\frac{v(80)-v(0)}{80-0} = \frac{49-5}{80} = \frac{11}{20}$ ft/sec^2	1 : answer
(b) Since the velocity is positive, $\int_{10}^{70} v(t)\,dt$ represents the distance, in feet, traveled by rocket A from $t = 10$ seconds to $t = 70$ seconds. A midpoint Riemann sum is $20[v(20)+v(40)+v(60)]$ $= 20[22+35+44] = 2020$ ft	3 : { 1 : explanation 1 : uses $v(20)$, $v(40)$, $v(60)$ 1 : value
(c) Let $v_B(t)$ be the velocity of rocket B at time t. $v_B(t) = \int \frac{3}{\sqrt{t+1}}\,dt = 6\sqrt{t+1} + C$ $2 = v_B(0) = 6 + C$ $v_B(t) = 6\sqrt{t+1} - 4$ $v_B(80) = 50 > 49 = v(80)$ Rocket B is traveling faster at time $t = 80$ seconds.	4 : { 1 : $6\sqrt{t+1}$ 1 : constant of integration 1 : uses initial condition 1 : finds $v_B(80)$, compares to $v(80)$, and draws a conclusion

2010 #2

(a) $E'(6) \approx \frac{E(7)-E(5)}{7-5} = 4$ hundred entries per hour	1 : answer
(b) $\frac{1}{8}\int_0^8 E(t)\,dt \approx$ $\frac{1}{8}\left(2\cdot\frac{E(0)+E(2)}{2} + 3\cdot\frac{E(2)+E(5)}{2} + 2\cdot\frac{E(5)+E(7)}{2} + 1\cdot\frac{E(7)+E(8)}{2}\right)$ $= 10.687$ or 10.688 $\frac{1}{8}\int_0^8 E(t)\,dt$ is the average number of hundreds of entries in the box between noon and 8 P.M.	3 : { 1 : trapezoidal sum 1 : approximation 1 : meaning
(c) $23 - \int_8^{12} P(t)\,dt = 23 - 16 = 7$ hundred entries	2 : { 1 : integral 1 : answer
(d) $P'(t) = 0$ when $t = 9.183503$ and $t = 10.816497$.	3 : { 1 : considers $P'(t) = 0$ 1 : identifies candidates 1 : answer with justification

t	$P(t)$
8	0
9.183503	5.088662
10.816497	2.911338
12	8

Entries are being processed most quickly at time $t = 12$.